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ELASTIC STRESSES IN AN INFINITE SOLID WITH A SPHERICAL CAVITY.

By T. H. Gronwall.

Sir Joseph Larmor has investigated the stresses in a solid having a cavity of dimensions very much smaller than those of the solid.* By a general physical argument, making no use of the mathematical expressions for the stress components, he arrives at the conclusion (l. c., p. 72) that the presence of the cavity materially affects the shearing stresses only, while in a column, for instance, acted upon by tensile or compressive external forces only, the tensile or compressive stresses are increased very slightly, viz., in the ratio of the cross section area of the cavity to that of the column. As will be shown below, this conclusion in respect to the tensile or compressive stresses is not borne out by the mathematical theory of elasticity.

Let us consider a solid extending to infinity in all directions, and having a spherical cavity of radius a with its center at the origin. Two cases of stress distribution at infinity are of special interest:

I. Pure shear:

$$X_x = Y_y = Z_z = Y_z = Z_x = 0,$$

$$X_y = S = \text{const.}, \text{ for } r^2 = x^2 + y^2 + z^2 = \infty;$$

II. Pure tension or compression:

$$X_x = Y_y = X_y = Y_z = Z_x = 0, \qquad Z_z = T = \text{const.}, \text{ for } r^2 = \infty.$$

The former case has been treated completely by Professor Love,† who derives the displacements u, v, w by the method of Lord Kelvin‡ and finds that at the surface of the cavity, for $x = y = 0$, $z = a$,

$$X_y = \frac{15\lambda + 30\mu}{9\lambda + 14\mu} S,$$

which is nearly twice as large as S.

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[*] The influence of flaws and air-cavities on the strength of materials, Philosophical Magazine, ser. 5, vol. 33 (1892), pp. 70–78.

[†] Larmor, l. c., pp. 76–78, and Love's Theory of Elasticity, 2d ed., § 178.

[‡] Phil. Trans. Roy. Soc., vol. 153 (1863); Math. and Phys. Papers, vol. 3 (Cambridge, 1890), p. 351.

In the case of pure tension or compression at infinity, I have applied the same method, obtaining the following values of the displacements:

$$u = \frac{a^3T}{2\mu}\left[-\frac{\lambda + 6\mu}{2(9\lambda + 14\mu)}\frac{x}{r^3} + \frac{\lambda + \mu}{9\lambda + 14\mu}(a^2 - r^2)\frac{\partial}{\partial x}\frac{3z^2 - r^2}{r^5} - \frac{\lambda}{3\lambda + 2\mu}\frac{x}{a^3}\right],$$

$$v = \frac{a^3T}{2\mu}\left[-\frac{\lambda + 6\mu}{2(9\lambda + 14\mu)}\frac{y}{r^3} + \frac{\lambda + \mu}{9\lambda + 14\mu}(a^2 - r^2)\frac{\partial}{\partial y}\frac{3z^2 - r^2}{r^5} - \frac{\lambda}{3\lambda + 2\mu}\frac{y}{a^3}\right],$$

$$w = \frac{a^3T}{2\mu}\left[\frac{11\lambda + 26\mu}{2(9\lambda + 14\mu)}\frac{z}{r^3} + \frac{\lambda + \mu}{9\lambda + 14\mu}(a^2 - r^2)\frac{\partial}{\partial z}\frac{3z^2 - r^2}{r^5} + \frac{2\lambda + 2\mu}{3\lambda + 2\mu}\frac{z}{a^3}\right].$$

The stress distribution is evidently one of revolution about the z-axis; calculating the stress components for $x = a$, $y = z = 0$, we find

$$X_x = 0, \qquad Y_y = -\frac{11\lambda + 6\mu}{18\lambda + 28\mu}T, \qquad Z_z = \frac{39\lambda + 54\mu}{18\lambda + 28\mu}T,$$
$$X_y = Y_z = Z_x = 0.$$

Thus at any point on the intersection of the sphere with the xy-plane, the stress Z_z is about twice as large as at infinity. This is in entire qualitative accordance with the result obtained by Kirsch in the corresponding plane problem: the presence of a circular hole in a plate of infinite dimensions makes the maximum tensile stress three times its value at infinity.